**Question-01**: What is Strum –Liouville problem?

**Solution:** Let , , and be real valued continuous functions of . Consider the boundary value problem

On some interval together with the boundary conditions

where is a parameter independent of and ,,, are given real constants such that at least one in each conditions (2) is different from zero.

The boundary value problem given by (1) and (2) is called Strum-Liouville problem.

**Question-02:** Construct Green’s function for the BVP and use it to find its solution.

**Solution:** The given boundary value problem is

and

Applying Laplace transform in (1) we get

Taking inverse Laplace transform we get

Using (2) in (3) we get

Putting this value in (3) we get

Now

Using (5) we get from (4)

where

This is the required Green’s function for the given problem.

**2nd part:** The solution is

where and

Now

And

Using these values we get from (6)

**(Ans)**

**Question-03:** Find the Green’s function of the BVP .

**Solution:** Given that

and

Let be the trial solution of (1). Then the auxiliary equation is

The general solution is

Using (2) in (3) we get

and

since, , so .

By putting the values of *A* and *B* in (3) we get

Since (1) has only trivial solution so the Green function exist and it is given by

where

The proposed Green’s function in (4) must satisfy the following three properties.

1. is continuous at , so we have

where is the coefficient of highest order derivative in (1).

1. must satisfy the boundary conditions (2).

Therefore and

and

and

From (5) and (6) by cross multiplication, we have

and

, since

and

By (9) we get from (8)

From (10) we have

Putting the values of , , ,and in (4), the required Green’s function is

**Question-04:** Using Green’s function solve the boundary value problem , .

**Solution:** Given that

and

The homogeneous form of (1) is

Let be the trial solution of (1). Then the auxiliary equation is

The complementary function is

By variation of parameter we can write

where and are functions of .

Putting

From (6) we get

Putting

From (7) and (10) we have

Using the values of and in (5) we get

The general solution is

The boundary conditions (2) gives

Putting the values of and in (12) we get

Now,

Putting this value in (13) we get

where

which is a Green function for the given problem.

**2nd part:**

**Question-05:** Construct Green’s function for the following boundary value problem:

**Solution:** The given boundary value problem is

and

Applying Laplace transform in (1) we get

Taking inverse Laplace transform we get

Using (2) in (3) we get

Putting this value in (3) we get

Now

Using (5) we get from (4)

where

This is the required Green’s function for the given problem.

**Question-06:** Using Green’s function solve the boundary value problem , .

**Solution:** Given that

and

The homogeneous form of (1) is

Let be the trial solution of (1). Then the auxiliary equation is

The complementary function is

By variation of parameter we can write

where and are functions of .

Putting

From (6) we get

Putting

From (7) and (10) we have

Using the values of and in (5) we get

The general solution is

The boundary conditions (2) gives

Putting the values of and in (12) we get

Now,

Putting this value in (13) we get

where

which is a Green function for the given problem.

**2nd part:**